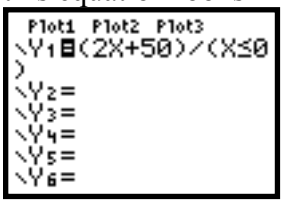
AP Calculus Summer Project

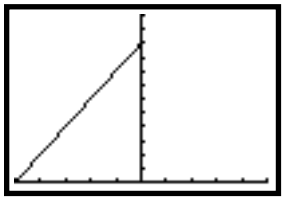
You don't want to forget calculus over the summer, do you? Don't answer that. Anyway, here is a project that will get your juices going and help you remember all about derivatives.

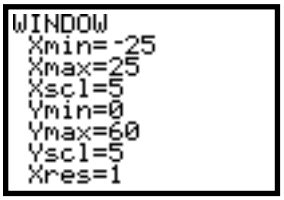
Your job is to design a roller coaster.

You will develop a piecewise curve with at least 5 pieces that is differentiable and continuous at all points.

Example. Suppose our roller coaster starts up the "lift ramp" defined by $y = 2x + 50$ when $x \le 0$. The graph of this equation looks like this:







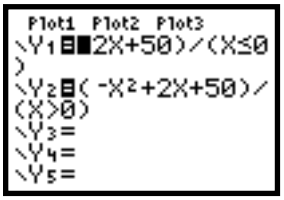
We need to develop a curve with is both continuous and differentiable at $x = 0$. Let's suppose it is a downward parabola in the form $f(x) = ax^2 + bx + c$.

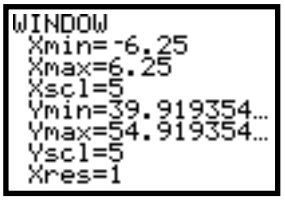
We know that in order to be continuous, than $f(0) = 50$, so $c = 50$, and $f(x) = ax^2 + bx + 50$.

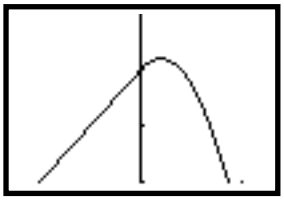
In order for $f(x)$ to be differentiable, then $f'(x) = 2ax + b$ must equal the slope of $2x + 50$ at $x = 0$. So $2ax + b = 2$ and when $x = 0$, $b = 2$.

So $f(x) = ax^2 + 2x + 50$. You can assign a to be any value you wish. Since it is to open downward, let $a = -1$.

So $f(x) = -1x^2 + 2x + 50$. Here is the graph and a zoom to show continuity and differentiability.







Here is another example: Suppose you have Y2 as $-x^2 + x + 40, x < 6$ and you wish to connect it with another parabola. So $x = 6$ is the point where your new parabola is going to begin.

Let us define our new parabola Y3 in the form $a(x-6)^2 + b(x-6) + c$. It will become apparent in a minute why I want to do that. Our goal is to find a, b, and c.

Now we want Y2 and Y3 to be continuous at $x = 6$.
Y2(6) = 10 by plugging in. Y3(6) = c. So $c = 10$. One piece of the puzzle solved.

Now we want Y2 and Y3 to be differentiable at $x = 6$.
Y2' = $-2x + 1$ so y2'(6) = -11. Y3' = $2a(x - 6) + b$ so Y3'(6) = b.
Since they must be the same, then $b = -11$. The second piece of the puzzle.
Now a can be anything you want. Since it will open up, a must be positive. For now, try $a = 1$.

So your Y3 = $1(x-6)^2 - 11(x-6) + 10$. Graph this for $x > 6$.

Note that this goes way below the axis. So play around with the value of *a*, making it much smaller, like 0.1. You can adjust it to anything you want. Remember though that the *x*-axis really means nothing. Don't think that the coaster has to start on the *x*-axis. To start it lower, just make your XMIN a smaller number. You might actually want to turn the axes of in your 2nd Format screen.

Hopefully you see why I chose the parabola in the $a(x-6)^2+b(x-6)+c$ format. It makes it far easier to work with because of the fact that when $x = 6$, many of the terms wash out. So if your 4th curve was a parabola which needed to start at $x = 10$, your Y4 = $a(x-10)^2+b(x-10)+c$ If it was a cube, it would be in the form $a(x-10)^3+b(x-10)^2+c(x-10)+d$. If it were an exponential, it would be ae^{x-10}
Finally, once you have your Y3, you may want to FOIL it out and collect terms so it looks "nicer."

In this project, you need at least 5 piecewise functions showing a possible roller coaster. You need to show the graph and a proof that the function you generate is continuous and differentiable at the points where the curves split into each piece. You may use any type of function we have discussed including linear, trig, exponential, logarithmic, inverse trig, and polynomial.

This is how you will present it: It is best to generate it on your calculator with 5(or more) piecewise functions. Once you have it complete, you will do the following.

1) Transfer it to a piece of poster paper. It can be drawn freehand but it should look like what you have on the calculator. Tape or glue several pieces of poster paper together horizontally. If you know how to use Excel, you may want to do the graphs in Excel, print it out, and then tape or glue it to the poster paper.

2) You will then cut out the roller coaster so that the top of it is your track.

3) Then tape or glue one end of the poster paper to the other end. You will end up with a 3-dimensional representation of your roller coaster.

4) Below each of your 5 (or more) curves, write its function. Use a heavy dot to show the transition points. On the poster paper itself, show that the curves are continuous and differentiable at those points.

5) Name your roller coaster.

Things to remember:

1) Your roller coaster should end at the same height it starts. You may want to have a horizontal line for your loading area. This does not count as one of the 5 curves.

2) Roller coasters never get as high as its previous high point. I will not take off for this but you may wish to keep it in mind.

3) Are you good enough to actually put in a loop? That would be spectacular.

These are due the first **full** class day of school in the fall. This will be a 20 point project. You will have several projects in AP Calculus 2 next year. So this is a way to get a head start on the year.

This is not an easy assignment. It will take time. You will present your roller coasters at the beginning of the year and they will be hung up in my classroom. So do a good job on them and don't let it go until the very end of the summer. Email me at the school website if you have problems or questions.